MATH 5300 - ASSIGNMENT 2

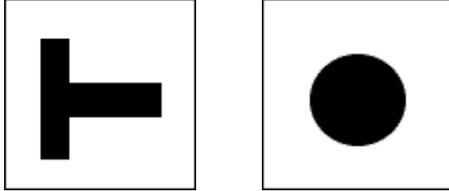MAY 7, 2008

to make this problem set (relatively) unique

(1) What are the first 4 four digits of your student id number? What is this number mod 8? mod 3? and mod 7? Use these numbers to do one part of question 2, 3, and 6 (which have 8, 3 and 7 parts respectively).

bit strings and logic

(1) Show that every binary function (there are 16) can be expressed as a composition of the binary operations AND/OR and NOT.
(2) Which (if any) of the 16 possible binary functions cannot be represented using only compositions of
    0. $\{AND, NOT\}$
    1. $\{AND, XOR\}$
    2. $\{OR, XOR\}$
    3. $\{NOT, XOR\}$
    4. $\{AND, IMPL\}$
    5. $\{OR, IMPL\}$
    6. $\{XOR, IMPL\}$
    7. $\{NOT, IMPL\}$.
(3) Which of the 16 binary operators are
    0. commutative?
    1. associative?
    2. idempotent (that is, when is $a \cdot a = a$ for $a = 0, 1$)?
(4) How do things grow/change if you were working in trinary? That is assume that there are three possible inputs $\{0, 1, 2\}$, how many possible binary (two input arguments $\rightarrow$ one output) operations are there?
(5) Again working with trinary inputs, how many binary operations are necessary so that compositions of these binary operations can be used to express every possible two argument function on trinary operations?

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(6) The two binary pictures listed below are pic1 and pic2.

Use GIMP or Photoshop or some other drawing program to create pictures (pic1 AND pic2) and (pic1 OR pic2). Also do one of (according to the first question):
  0. pic1 XOR pic2
  1. pic1 IMPL pic2
  2. (NOT pic1) AND (NOT pic2)
  3. (NOT pic1) XOR pic2
  4. NOT (pic1 IMPL pic2)
  5. (NOT pic1) IMPL (NOT pic2)
  6. pic1 AND (pic1 IMPL pic2)
These pictures can be downloaded off the wiki for math 5300 so it is not necessary to redraw them yourself.

(7) Choose symbols to represent the truth values of phrases which appear in the following paragraph which cannot be decomposed in terms of the operations of AND, OR and NOT. Join these symbols together with the three connectives and represent each sentence with connectives and symbols. (example: "The murder was committed by at least one of the staff– the butler, the maid and the gardner" can be represented by "A OR B OR C" where A = the butler committed the murder, B = the maid did it, C = the gardener did it)

Poison caused the victim's death if and only if there was a change in his blood chemistry or a residue of poison in the stomach. There was neither a change in blood chemistry nor a residue of poison in his stomach, but there were puncture marks on the body. Poison was injected by a needle only if there were puncture marks on the body. Either poison was the cause of the victim's death, or there are no puncture marks on the body.

orders of magnitude

(1) A Commodore 64 computer boasted 64K of memory in its day. How many times more memory does a computer with 1GB of memory have today?

(2) A double sided floppy disk (the 5 and a quarter inch kind) could hold 800K of storage. How many floppy disks are equivalent to DVDs which can store 4.7GB of data?

(3) Estimate processor clock rates of early computers by finding information on the internet (reference your source). Estimate also some typical computers on the market today and compare factors of speed.